

Formula Sheet for FOUNDATION MATHEMATICS B (H8006Z)

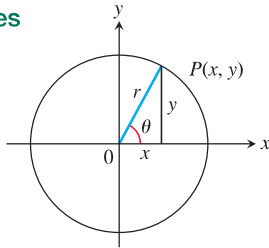
Trigonometry Formulas

Definitions and Fundamental Identities

Sine: $\sin \theta = \frac{y}{r} = \frac{1}{\csc \theta}$

Cosine: $\cos \theta = \frac{x}{r} = \frac{1}{\sec \theta}$

Tangent: $\tan \theta = \frac{y}{x} = \frac{1}{\cot \theta}$



Identities

$\sin(-\theta) = -\sin \theta, \quad \cos(-\theta) = \cos \theta$

$\sin^2 \theta + \cos^2 \theta = 1, \quad \sec^2 \theta = 1 + \tan^2 \theta, \quad \csc^2 \theta = 1 + \cot^2 \theta$

$\sin 2\theta = 2 \sin \theta \cos \theta, \quad \cos 2\theta = \cos^2 \theta - \sin^2 \theta$

$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}, \quad \sin^2 \theta = \frac{1 - \cos 2\theta}{2}$

$\sin(A + B) = \sin A \cos B + \cos A \sin B$

$\sin(A - B) = \sin A \cos B - \cos A \sin B$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

$$\sin\left(A - \frac{\pi}{2}\right) = -\cos A, \quad \cos\left(A - \frac{\pi}{2}\right) = \sin A$$

$$\sin\left(A + \frac{\pi}{2}\right) = \cos A, \quad \cos\left(A + \frac{\pi}{2}\right) = -\sin A$$

$$\sin A \sin B = \frac{1}{2} \cos(A - B) - \frac{1}{2} \cos(A + B)$$

$$\cos A \cos B = \frac{1}{2} \cos(A - B) + \frac{1}{2} \cos(A + B)$$

$$\sin A \cos B = \frac{1}{2} \sin(A - B) + \frac{1}{2} \sin(A + B)$$

$$\sin A + \sin B = 2 \sin \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$$

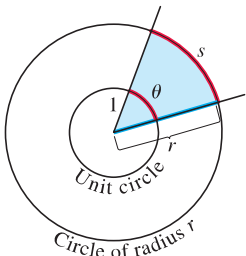
$$\sin A - \sin B = 2 \cos \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)$$

$$\cos A + \cos B = 2 \cos \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$$

$$\cos A - \cos B = -2 \sin \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)$$

Trigonometric Functions

Radian Measure

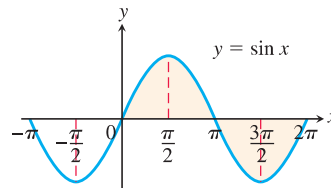


$$\frac{s}{r} = \frac{\theta}{1} = \theta \quad \text{or} \quad \theta = \frac{s}{r}$$

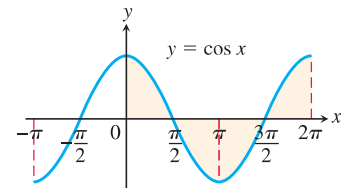
$$180^\circ = \pi \text{ radians.}$$

Degrees	Radians

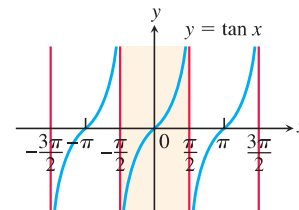
The angles of two common triangles, in degrees and radians.



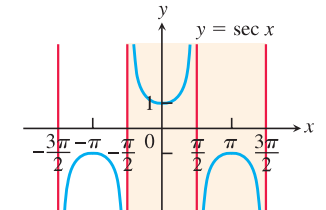
Domain: $(-\infty, \infty)$
Range: $[-1, 1]$



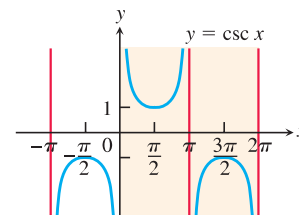
Domain: $(-\infty, \infty)$
Range: $[-1, 1]$



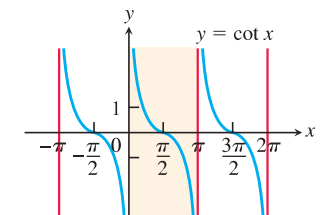
Domain: All real numbers except odd integer multiples of $\pi/2$
Range: $(-\infty, \infty)$



Domain: All real numbers except odd integer multiples of $\pi/2$
Range: $(-\infty, -1] \cup [1, \infty)$



Domain: $x \neq 0, \pm\pi, \pm2\pi, \dots$
Range: $(-\infty, -1] \cup [1, \infty)$



Domain: $x \neq 0, \pm\pi, \pm2\pi, \dots$
Range: $(-\infty, \infty)$

Series

Tests for Convergence of Infinite Series

- 1. The n th-Term Test:** Unless $a_n \rightarrow 0$, the series diverges.
- 2. Geometric series:** $\sum ar^n$ converges if $|r| < 1$; otherwise it diverges.
- 3. p -series:** $\sum 1/n^p$ converges if $p > 1$; otherwise it diverges.
- 4. Series with nonnegative terms:** Try the Integral Test, Ratio Test, or Root Test. Try comparing to a known series with the Comparison Test or the Limit Comparison Test.
- 5. Series with some negative terms:** Does $\sum |a_n|$ converge? If yes, so does $\sum a_n$ because absolute convergence implies convergence.
- 6. Alternating series:** $\sum a_n$ converges if the series satisfies the conditions of the Alternating Series Test.

Taylor Series

$$\frac{1}{1-x} = 1 + x + x^2 + \cdots + x^n + \cdots = \sum_{n=0}^{\infty} x^n, \quad |x| < 1$$

$$\frac{1}{1+x} = 1 - x + x^2 - \cdots + (-x)^n + \cdots = \sum_{n=0}^{\infty} (-1)^n x^n, \quad |x| < 1$$

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \quad |x| < \infty$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}, \quad |x| < \infty$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n \frac{x^{2n}}{(2n)!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}, \quad |x| < \infty$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n-1} \frac{x^n}{n} + \cdots = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} x^n}{n}, \quad -1 < x \leq 1$$

$$\ln \frac{1+x}{1-x} = 2 \tanh^{-1} x = 2 \left(x + \frac{x^3}{3} + \frac{x^5}{5} + \cdots + \frac{x^{2n+1}}{2n+1} + \cdots \right) = 2 \sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1}, \quad |x| < 1$$

$$\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots + (-1)^n \frac{x^{2n+1}}{2n+1} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}, \quad |x| \leq 1$$

Binomial Series

$$\begin{aligned} (1+x)^m &= 1 + mx + \frac{m(m-1)x^2}{2!} + \frac{m(m-1)(m-2)x^3}{3!} + \cdots + \frac{m(m-1)(m-2)\cdots(m-k+1)x^k}{k!} + \cdots \\ &= 1 + \sum_{k=1}^{\infty} \binom{m}{k} x^k, \quad |x| < 1, \end{aligned}$$

where

$$\binom{m}{1} = m, \quad \binom{m}{2} = \frac{m(m-1)}{2!}, \quad \binom{m}{k} = \frac{m(m-1)\cdots(m-k+1)}{k!} \quad \text{for } k \geq 3.$$

Fourier Series

A function $f(t)$ with period 2π has a Fourier series

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos nt + b_n \sin nt)$$

The Fourier coefficients are

$$\begin{aligned} a_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos nt \, dt \quad n = 0, 1, 2, \dots \\ b_n &= \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin nt \, dt \quad n = 1, 2, \dots \end{aligned}$$

In the integrals any convenient integration range extending over an interval of 2π may be used.

Fourier series for functions of general period

A function $f(t)$ with period T has a Fourier series

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left\{ a_n \cos\left(\frac{2n\pi t}{T}\right) + b_n \sin\left(\frac{2n\pi t}{T}\right) \right\}$$

The Fourier coefficients are

$$a_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \cos\left(\frac{2n\pi t}{T}\right) dt \quad n = 0, 1, 2, \dots$$

$$b_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \sin\left(\frac{2n\pi t}{T}\right) dt \quad n = 1, 2, \dots$$

Vector Operator Formulas (Cartesian Form)

Formulas for Grad, Div, Curl, and the Laplacian

Cartesian (x, y, z) \mathbf{i} , \mathbf{j} , and \mathbf{k} are unit vectors in the directions of increasing x , y , and z . M , N , and P are the scalar components of $\mathbf{F}(x, y, z)$ in these directions.

Gradient $\nabla f = \frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} + \frac{\partial f}{\partial z} \mathbf{k}$

Divergence $\nabla \cdot \mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

Curl $\nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix}$

Laplacian $\nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$

Vector Triple Products

$$(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w} = (\mathbf{v} \times \mathbf{w}) \cdot \mathbf{u} = (\mathbf{w} \times \mathbf{u}) \cdot \mathbf{v}$$

$$\mathbf{u} \times (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \cdot \mathbf{w})\mathbf{v} - (\mathbf{u} \cdot \mathbf{v})\mathbf{w}$$

Vector Identities

In the identities here, f and g are differentiable scalar functions; \mathbf{F} , \mathbf{F}_1 , and \mathbf{F}_2 are differentiable vector fields; and a and b are real constants.

$$\nabla \times (\nabla f) = \mathbf{0}$$

$$\nabla(fg) = f\nabla g + g\nabla f$$

$$\nabla \cdot (g\mathbf{F}) = g\nabla \cdot \mathbf{F} + \nabla g \cdot \mathbf{F}$$

$$\nabla \times (g\mathbf{F}) = g\nabla \times \mathbf{F} + \nabla g \times \mathbf{F}$$

$$\nabla \cdot (a\mathbf{F}_1 + b\mathbf{F}_2) = a\nabla \cdot \mathbf{F}_1 + b\nabla \cdot \mathbf{F}_2$$

$$\nabla \times (a\mathbf{F}_1 + b\mathbf{F}_2) = a\nabla \times \mathbf{F}_1 + b\nabla \times \mathbf{F}_2$$

$$\nabla(\mathbf{F}_1 \cdot \mathbf{F}_2) = (\mathbf{F}_1 \cdot \nabla)\mathbf{F}_2 + (\mathbf{F}_2 \cdot \nabla)\mathbf{F}_1 +$$

$$\mathbf{F}_1 \times (\nabla \times \mathbf{F}_2) + \mathbf{F}_2 \times (\nabla \times \mathbf{F}_1)$$

$$\nabla \cdot (\mathbf{F}_1 \times \mathbf{F}_2) = \mathbf{F}_2 \cdot (\nabla \times \mathbf{F}_1) - \mathbf{F}_1 \cdot (\nabla \times \mathbf{F}_2)$$

$$\nabla \times (\mathbf{F}_1 \times \mathbf{F}_2) = (\mathbf{F}_2 \cdot \nabla)\mathbf{F}_1 - (\mathbf{F}_1 \cdot \nabla)\mathbf{F}_2 +$$

$$(\nabla \cdot \mathbf{F}_2)\mathbf{F}_1 - (\nabla \cdot \mathbf{F}_1)\mathbf{F}_2$$

$$\nabla \times (\nabla \times \mathbf{F}) = \nabla(\nabla \cdot \mathbf{F}) - (\nabla \cdot \nabla)\mathbf{F} = \nabla(\nabla \cdot \mathbf{F}) - \nabla^2 \mathbf{F}$$

$$(\nabla \times \mathbf{F}) \times \mathbf{F} = (\mathbf{F} \cdot \nabla)\mathbf{F} - \frac{1}{2}\nabla(\mathbf{F} \cdot \mathbf{F})$$

The Fundamental Theorem of Line Integrals

Part 1 Let $\mathbf{F} = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ be a vector field whose components are continuous throughout an open connected region D in space. Then there exists a differentiable function f such that

$$\mathbf{F} = \nabla f = \frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} + \frac{\partial f}{\partial z} \mathbf{k}$$

if and only if for all points A and B in D , the value of $\int_A^B \mathbf{F} \cdot d\mathbf{r}$ is independent of the path joining A to B in D .

Part 2 If the integral is independent of the path from A to B , its value is

$$\int_A^B \mathbf{F} \cdot d\mathbf{r} = f(B) - f(A).$$

Green's Theorem and Its Generalization to Three Dimensions

Tangential form of Green's Theorem: $\oint_C \mathbf{F} \cdot \mathbf{T} ds = \iint_R (\nabla \times \mathbf{F}) \cdot \mathbf{k} dA$

Stokes' Theorem: $\oint_C \mathbf{F} \cdot \mathbf{T} ds = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} d\sigma$

Normal form of Green's Theorem: $\oint_C \mathbf{F} \cdot \mathbf{n} ds = \iint_R (\nabla \cdot \mathbf{F}) dA$

Divergence Theorem: $\iint_S \mathbf{F} \cdot \mathbf{n} d\sigma = \iiint_D \nabla \cdot \mathbf{F} dV$

Limits

General Laws

If L, M, c , and k are real numbers and

$$\lim_{x \rightarrow c} f(x) = L \quad \text{and} \quad \lim_{x \rightarrow c} g(x) = M, \quad \text{then}$$

Sum Rule: $\lim_{x \rightarrow c} (f(x) + g(x)) = L + M$

Difference Rule: $\lim_{x \rightarrow c} (f(x) - g(x)) = L - M$

Product Rule: $\lim_{x \rightarrow c} (f(x) \cdot g(x)) = L \cdot M$

Constant Multiple Rule: $\lim_{x \rightarrow c} (k \cdot f(x)) = k \cdot L$

Quotient Rule: $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{L}{M}, \quad M \neq 0$

The Sandwich Theorem

If $g(x) \leq f(x) \leq h(x)$ in an open interval containing c , except possibly at $x = c$, and if

$$\lim_{x \rightarrow c} g(x) = \lim_{x \rightarrow c} h(x) = L,$$

then $\lim_{x \rightarrow c} f(x) = L$.

Inequalities

If $f(x) \leq g(x)$ in an open interval containing c , except possibly at $x = c$, and both limits exist, then

$$\lim_{x \rightarrow c} f(x) \leq \lim_{x \rightarrow c} g(x).$$

Continuity

If g is continuous at L and $\lim_{x \rightarrow c} f(x) = L$, then

$$\lim_{x \rightarrow c} g(f(x)) = g(L).$$

Specific Formulas

If $P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_0$, then

$$\lim_{x \rightarrow c} P(x) = P(c) = a_n c^n + a_{n-1} c^{n-1} + \cdots + a_0.$$

If $P(x)$ and $Q(x)$ are polynomials and $Q(c) \neq 0$, then

$$\lim_{x \rightarrow c} \frac{P(x)}{Q(x)} = \frac{P(c)}{Q(c)}.$$

If $f(x)$ is continuous at $x = c$, then

$$\lim_{x \rightarrow c} f(x) = f(c).$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad \text{and} \quad \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$

L'Hôpital's Rule

If $f(a) = g(a) = 0$, both f' and g' exist in an open interval I containing a , and $g'(x) \neq 0$ on I if $x \neq a$, then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)},$$

assuming the limit on the right side exists.

Differentiation Rules

General Formulas

Assume u and v are differentiable functions of x .

Constant: $\frac{d}{dx}(c) = 0$

Sum: $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$

Difference: $\frac{d}{dx}(u - v) = \frac{du}{dx} - \frac{dv}{dx}$

Constant Multiple: $\frac{d}{dx}(cu) = c \frac{du}{dx}$

Product: $\frac{d}{dx}(uv) = u \frac{dv}{dx} + \frac{du}{dx} v$

Quotient: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$

Power: $\frac{d}{dx}x^n = nx^{n-1}$

Chain Rule: $\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$

Trigonometric Functions

$$\frac{d}{dx}(\sin x) = \cos x \quad \frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x \quad \frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x \quad \frac{d}{dx}(\csc x) = -\csc x \cot x$$

Exponential and Logarithmic Functions

$$\frac{d}{dx}e^x = e^x \quad \frac{d}{dx} \ln x = \frac{1}{x}$$

$$\frac{d}{dx}a^x = a^x \ln a \quad \frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

Inverse Trigonometric Functions

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}} \quad \frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2} \quad \frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2} \quad \frac{d}{dx}(\csc^{-1} x) = -\frac{1}{|x|\sqrt{x^2-1}}$$

Hyperbolic Functions

$$\frac{d}{dx}(\sinh x) = \cosh x \quad \frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x \quad \frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$$

$$\frac{d}{dx}(\coth x) = -\operatorname{csch}^2 x \quad \frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$$

Inverse Hyperbolic Functions

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}} \quad \frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2} \quad \frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2} \quad \frac{d}{dx}(\operatorname{csch}^{-1} x) = -\frac{1}{|x|\sqrt{1+x^2}}$$

Parametric Equations

If $x = f(t)$ and $y = g(t)$ are differentiable, then

$$y' = \frac{dy}{dx} = \frac{dy/dt}{dx/dt} \quad \text{and} \quad \frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt}$$

Integration Rules

General Formulas

Zero:
$$\int_a^a f(x) dx = 0$$

Order of Integration:
$$\int_b^a f(x) dx = -\int_a^b f(x) dx$$

Constant Multiples:
$$\int_a^b kf(x) dx = k \int_a^b f(x) dx, \quad k \text{ any number}$$

$$\int_a^b -f(x) dx = -\int_a^b f(x) dx, \quad k = -1$$

Sums and Differences:
$$\int_a^b (f(x) \pm g(x)) dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$$

Additivity:
$$\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$$

Max-Min Inequality: If $\max f$ and $\min f$ are the maximum and minimum values of f on $[a, b]$, then

$$\min f \cdot (b - a) \leq \int_a^b f(x) dx \leq \max f \cdot (b - a).$$

Domination: $f(x) \geq g(x)$ on $[a, b]$ implies $\int_a^b f(x) dx \geq \int_a^b g(x) dx$

$$f(x) \geq 0 \text{ on } [a, b] \text{ implies } \int_a^b f(x) dx \geq 0$$

The Fundamental Theorem of Calculus

Part 1 If f is continuous on $[a, b]$, then $F(x) = \int_a^x f(t) dt$ is continuous on $[a, b]$ and differentiable on (a, b) and its derivative is $f(x)$:

$$F'(x) = \frac{d}{dx} \int_a^x f(t) dt = f(x).$$

Part 2 If f is continuous at every point of $[a, b]$ and F is any antiderivative of f on $[a, b]$, then

$$\int_a^b f(x) dx = F(b) - F(a).$$

Substitution in Definite Integrals

$$\int_a^b f(g(x)) \cdot g'(x) dx = \int_{g(a)}^{g(b)} f(u) du$$

Integration by Parts

$$\int_a^b u(x) v'(x) dx = u(x) v(x) \Big|_a^b - \int_a^b v(x) u'(x) dx$$

A Brief Table of Integrals follows the Index at the back of the text.

1. $\int k \, dx = kx + C$ (any number k)
2. $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$ ($n \neq -1$)
3. $\int \frac{dx}{x} = \ln |x| + C$
4. $\int e^x \, dx = e^x + C$
5. $\int a^x \, dx = \frac{a^x}{\ln a} + C$ ($a > 0, a \neq 1$)
6. $\int \sin x \, dx = -\cos x + C$
7. $\int \cos x \, dx = \sin x + C$
8. $\int \sec^2 x \, dx = \tan x + C$
8. $\int \csc^2 x \, dx = -\cot x + C$
10. $\int \sec x \tan x \, dx = \sec x + C$
11. $\int \csc x \cot x \, dx = -\csc x + C$
12. $\int \tan x \, dx = \ln |\sec x| + C$
13. $\int \cot x \, dx = \ln |\sin x| + C$
14. $\int \sec x \, dx = \ln |\sec x + \tan x| + C$
15. $\int \csc x \, dx = -\ln |\csc x + \cot x| + C$
16. $\int \sinh x \, dx = \cosh x + C$
17. $\int \cosh x \, dx = \sinh x + C$
18. $\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$
19. $\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$
20. $\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{a} \sec^{-1}\left|\frac{x}{a}\right| + C$
21. $\int \frac{dx}{\sqrt{a^2 + x^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C$ ($a > 0$)
22. $\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C$ ($x > a > 0$)

END

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$\frac{1}{s}$	2. e^{at}	$\frac{1}{s-a}$
3. $t^n, n = 1, 2, 3, \dots$	$\frac{n!}{s^{n+1}}$	4. $t^p, p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5. \sqrt{t}	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6. $t^{n-\frac{1}{2}}, n = 1, 2, 3, \dots$	$\frac{1 \cdot 3 \cdot 5 \cdots (2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7. $\sin(at)$	$\frac{a}{s^2+a^2}$	8. $\cos(at)$	$\frac{s}{s^2+a^2}$
9. $t \sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10. $t \cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11. $\sin(at) - at \cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12. $\sin(at) + at \cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13. $\cos(at) - at \sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14. $\cos(at) + at \sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15. $\sin(at+b)$	$\frac{s \sin(b) + a \cos(b)}{s^2+a^2}$	16. $\cos(at+b)$	$\frac{s \cos(b) - a \sin(b)}{s^2+a^2}$
17. $\sinh(at)$	$\frac{a}{s^2-a^2}$	18. $\cosh(at)$	$\frac{s}{s^2-a^2}$
19. $e^{at} \sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20. $e^{at} \cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21. $e^{at} \sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22. $e^{at} \cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23. $t^n e^{at}, n = 1, 2, 3, \dots$	$\frac{n!}{(s-a)^{n+1}}$	24. $f(ct)$	$\frac{1}{c} F\left(\frac{s}{c}\right)$
25. $u_c(t) = u(t-c)$	$\frac{e^{-cs}}{s}$	26. $\delta(t-c)$	e^{-cs}
27. $u_c(t)f(t-c)$	$e^{-cs}F(s)$	28. $u_c(t)g(t)$	$e^{-cs}\mathcal{L}\{g(t+c)\}$
29. $e^{ct}f(t)$	$F(s-c)$	30. $t^n f(t), n = 1, 2, 3, \dots$	$(-1)^n F^{(n)}(s)$
31. $\frac{1}{t}f(t)$	$\int_s^\infty F(u) du$	32. $\int_0^t f(v) dv$	$\frac{F(s)}{s}$
33. $\int_0^t f(t-\tau)g(\tau) d\tau$	$F(s)G(s)$	34. $f(t+T) = f(t)$	$\frac{\int_0^T e^{-st} f(t) dt}{1-e^{-sT}}$
35. $f'(t)$	$sF(s) - f(0)$	36. $f''(t)$	$s^2F(s) - sf(0) - f'(0)$
37. $f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - s^{n-2}f'(0) \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0)$		